Design of Circular Beam Example

ACI 318-99

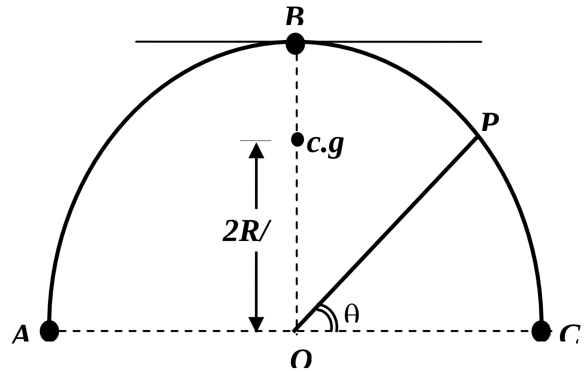
Dr. Mohammed Arafa

Design of Circular Beam
Example 1 (ACI 318-99)

Design a semi-circular beam supported on three-equally spaced columns. The centers of the columns are on a circular curve of diameter 8m. The beam is support a uniformly distributed factored load of 5.0 t/m, in addition to its own weight.

Use $f_c' = 350\,kg/cm^2$ and $f_y = 4200\,kg/cm^2$

Solution



$L = r(\pi/2) = 6.28\,m$

$h_{\min} = L/18.5 = 628/18.5 = 33.94\,cm$

use Beam 40×70 cm

o. w. of the beam =0.4(0.7)(2.5)(1.4)=0.98 t/m’

total load= 5.14+0.84=5.98 t/m’

$M_{\max(-ve)} = 0.429wR^2 = 0.429(5.98)(4)^2 = 41.05\,t.m$

$M_{\max(+ve)} = 0.1514wR^2 = 0.1514(5.98)(4)^2 = 14.48\,t.m$

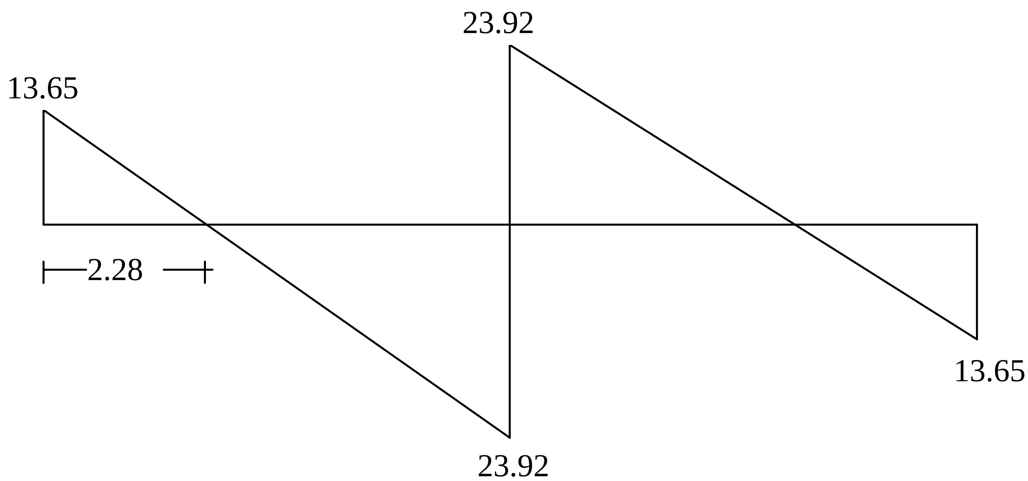
$T_{\max} = 0.1042wR^2 = 0.1042(5.98)(4)^2 = 9.97\,t.m$

Reactions

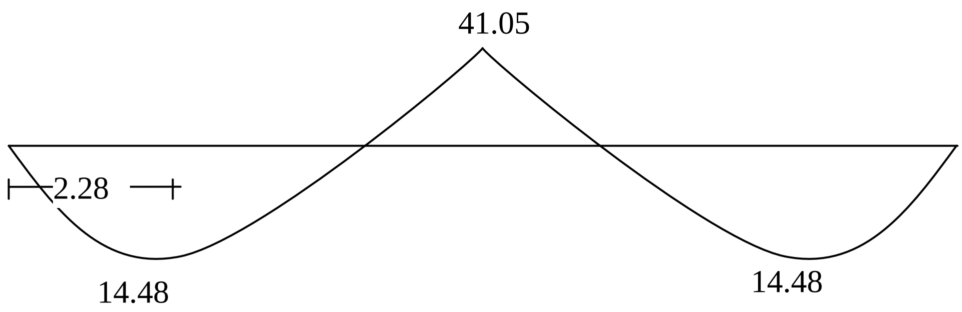
$R_A = \frac{wR}{2}(\pi - 2) = \frac{5.98(4)}{2}(\pi - 2) = 13.65\,ton$

$R_B = 2wR = 2(5.98)(4) = 47.84\,ton$

Shear at point P: $V_\theta = \frac{wR}{2}(\pi - 2) - wR\theta$



Shear Force Diagram



Bending Moment Diagram

Design for Reinforcement

d=70-4.0-0.8-1.25=63.95

$$\rho_{-ve} = \frac{0.85 \cdot 350}{4200}\left[1-\sqrt{1-\frac{2.61 \cdot 10^5 (41.05)}{40 \cdot 63.95 \cdot 350}}\right] = 0.0697 > \rho_{\min}$$

$$A_{s(-ve)} = (0.00697)(40)(63.95) = 17.85 cm^2$$

$$\rho_{-ve} = \frac{0.85 \cdot 350}{4200}\left[1-\sqrt{1-\frac{2.61 \cdot 10^5 (14.48)}{40 \cdot 63.95 \cdot 350}}\right] = 0.00238 < \rho_{\min}$$

$$A_{s(+ve)} = (0.0033)(40)(63.95) = 8.44 cm^2$$

Design for Shear

$V_{\max} = 23.92 ton$ ($T = 0.0$ at middle support)

$$\phi V_c = 0.53(0.85)\sqrt{350}(40)(63.95) = 21.56 ton$$

$$V_S = \frac{23.92 - 21.56}{0.85} = 2.77\ ton$$

$$2.77 \cdot 1000 = \frac{A_V}{S} 4200(63.95)$$

$$\frac{A_V}{S} = 0.0103\ cm^2/cm$$

Design for Torsion

$$T_u \le \frac{0.265\phi\sqrt{f_c'}A_{cp}^2}{p_{cp}} \Rightarrow \quad \text{neglect torsion}$$

$A_{cp} = b\,h$, $p_{cp} = 2(b+h)$, $A_{0h} = x_0 y_0$, $p_h = 2(x_0 + y_0)$ and $A_0 = 0.85 x_0 y_0$

The following equation has to be satisfied to check for ductlity

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)} \le 0.263\phi\sqrt{f_c'}$$

Reinforcement

$$\frac{A_T}{S} = \frac{T_u}{2\phi f_{ys} A_0}$$

$$A_l = \left(\frac{A_T}{S}\right)p_h \quad \text{and} \quad A_{l,\min} = \frac{1.3\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_T}{S}\right)p_h$$

At section of maximum torsion is located at θ=59.43

T=9.97 t.m

$$V_u = \frac{wR}{2}(\pi - 2) - wR\theta = 13.65 - 5.98(4)(\frac{59.43}{360}2\pi) = -11.6\,ton$$

$M = 0.0$

$$\phi V_c = 0.53(0.85)\sqrt{350}(40)(63.95) = 21.56\,ton > V_u \quad (\text{No shear reinf. required})$$

$$\frac{0.265\phi\sqrt{f_c'}A_{cp}^2}{p_{cp}} = \frac{0.265(0.85)\sqrt{f_c'}\,(40\cdot 70)^2}{2(40+70)10^5} = 1.50\,t.m < T_u$$

i.e torsion must be considered

Ductility Check

The following equation has to be satisfied

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)} \le 0.263\phi\sqrt{f_c^{'}}$$

$$x_0 = 40-4-4-0.8 = 31.2$$

$$y_0 = 70-4-4-0.8 = 61.2$$

$$p_h = 2(x_0+y_0) = 2(31.2+61.2) = 184.8cm$$

$$A_{oh} = 31.2\cdot 61.2 = 1909.44\ cm^2$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)} = \sqrt{\left(\frac{11.16\cdot 10^3}{40\cdot 63.95}\right)^2+\left(\frac{(9.97\cdot 10^5)(184.8)}{1.7(1909.44)^2}\right)} = 30.04\ kg\ /cm^2$$

$$0.263\phi\sqrt{f_c^{'}} = 0.263(0.85)\sqrt{350} = 41.82\ kg\ /cm^2 > 30.04\ kg\ /cm^2$$

i.e section dimension are adequate for preventing brittle failure due to combined shear stresses.

$$\frac{A_T}{S} = \frac{T_u}{2\phi f_{ys}A_0} = \frac{9.97\cdot 10^5}{2(0.85)4200(0.85\cdot 1909.44)} = 0.086\ cm^2\ /cm$$

$$\left(\frac{A_T}{S}\right)_{\min} = \frac{3.5b_w}{2f_{ys}} = \frac{3.5(40)}{2(4200)} = 0.0167cm^2\ /cm$$

$$\left(\frac{A_V}{S}\right)+\left(\frac{A_T}{S}\right) = 0+0.086 = 0.086cm^2\ /cm$$

Use ϕ12mm @12.5cm (closed stirrups) with $\frac{\text{A}}{\text{S}} = \frac{1.13}{12.5} = 0.0904cm^2\ /cm$

$$A_l = \left(\frac{A_T}{S}\right)p_h = 0.086(184.8) = 15.98cm^2$$

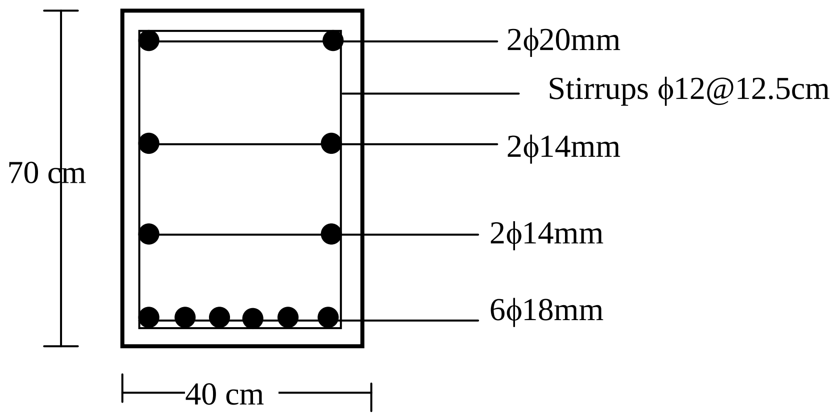
$$A_{l,\min} = \frac{1.3\sqrt{f_c^{'}}A_{cp}}{f_y}-\left(\frac{A_T}{S}\right)p_h = \frac{1.3\sqrt{350}\,(40\cdot 70)}{4200}-15.98 = 0.234cm^2$$

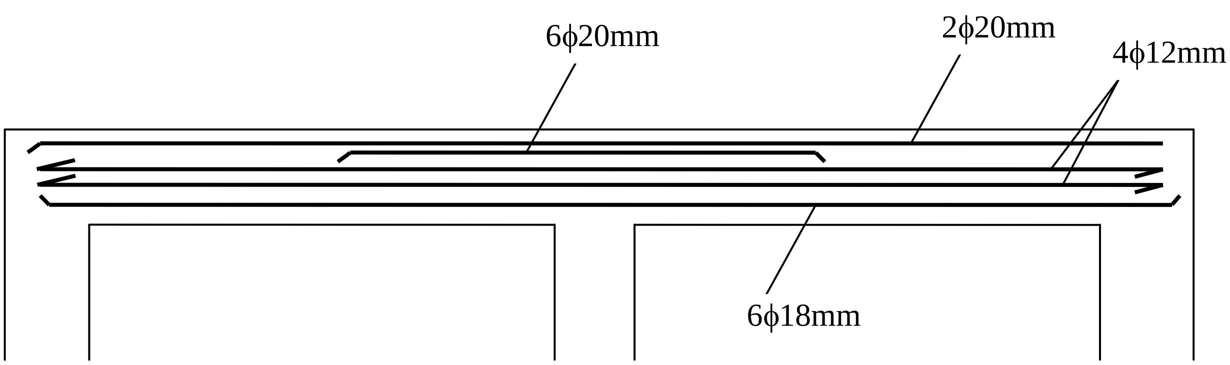
$$A_l = \frac{15.98}{3} = 5.33cm^2$$

Use $2\phi 20$ top and $4\phi 14$ skin reinforcement

$(A_{s,+ve})_{total} = 5.33+8.44 = 13.77cm^2$ use 6ϕ18mm

$(A_{s,-ve})_{total} = 17.83+8.44 = 23.16cm^2$ use 8ϕ20mm


2ϕ20mm
Stirrups ϕ12@12.5cm
2ϕ14mm
70 cm
2ϕ14mm
6ϕ18mm
40 cm


6ϕ20mm
2ϕ20mm
4ϕ12mm
6ϕ18mm